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PROBLEM SOLVING IN GEOMETRY

Jumaniyozov Kudrat Sapoyevich
Candidate of Pedagogical Sciences (PhD), Associate Professor,
Head of the Department "Methods of exact and natural sciences" of
Tashkent regional center of training and in-service of public education stuff

Abstract.This article is aimed at developing school students' interest in geometry classes, developing their imagination in solving geometric problems.
Keywords.geometry, problem, algebraic method, skill, theorem, verbal methods, geometric shapes, mathematical questions, thinking.

INTRODUCTION

In order to solve geometric problems, its condition is analyzed in depth and a form corresponding to the analysis is created. For example, it is impossible to learn to solve geometric problems without creating a shape that fits the condition, without developing certain skills about it. It should be a habit not to start solving a problem without creating a form that satisfies mathematical requirements and is aesthetically pleasing.

The algebraic method is widely used in solving geometric problems. The advantage of this method over other methods is that it can be algorithmically. Each geometric problem is solved by an elementary problem. Problems that can be solved in a single operation (step) using certain theorems and formulas are called simple (elementary) problems.

Thus, the development of students' ability to solve geometric problems can be determined using two factors - the shape they make and the method they use (primarily the algebraic method). It also requires knowledge of some additional geometric facts and commonly used reference issues when solving problems. The school can be seen as the underlying problem of many theorems given as problems in a geometry course.

In the school geometry course, metric problems are solved mainly using cosines, sine, and Pythagorean theorems.

The cosine theorem identifies the following three elementary problems:

1.Given the two sides of a triangle and the angle between them. Let's find the third side.

2.Given three sides of a triangle. Find the arbitrary angle of the triangle.

3.Given two sides of a triangle and an angle that does not lie between them. Let us find the third side of the triangle.

If the element sought in the first two problems is found to have the same value, in the third problem, in most cases it is necessary to solve the quadratic equation.

1st Problem. The sides of a triangle form an arithmetic progression with a difference of 3. If the smallest corner of a triangle has a cosine, find its perimeter. $\frac{7}{8}$

Solution. We define the middle side of the triangle as x in relation to the length of the side. In that case the remaining sides are x-3 and x + 3.

given α according to the cosines theorem because the angle is opposite to the x-3 side

$$(x-3)^2 = x^2 + (x+3)^2 - 2x(x+3)cos\alpha$$

We have the equation. In this case x = 9. The answer is P = 27

Now let's look at a non-elementary issue.

2nd Problem. The sides of the triangle are AB = 6, BC = 10, and AC = 12. The point P is on the AB side AP: PB = 1: 2 and the F point is on the BC side BF: FC = 2: 3. Find the cross-sectional length of PF (Figure 1).

Given:

AB = 6

BC = 10

AC = 12

AP: PB = 1: 2

BM: MC = 2: 3

Need to find PF =?

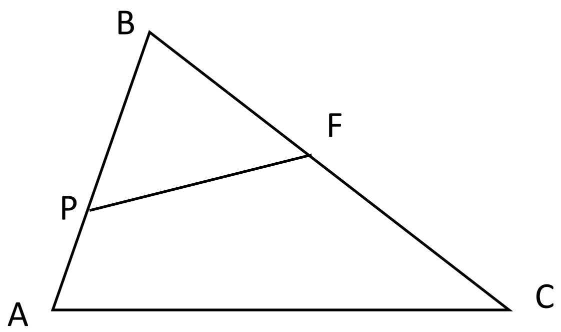


Figure 1

The solution to the problem consists of two steps. In the first step, the cosine of angle B is found based on the cosines theorem. The second step is to find the PF cross section from the BPF triangle again according to the cosines theorem.

Answer: $8\sqrt{\frac{2}{15}}$

In many cases, students are unable to see the right-angled triangle in the problem condition, resulting in difficulty in solving it. Therefore, we consider the "right triangle" separately and give the following affirmations:

1. The height reduced to the hypotenuse of a right-angled triangle divides it into mutually similar triangles.

2. If the cross section CD is the height of the triangle ABC reduced to the hypotenuse AB, then the relation CD2 = AD * DB, AC2 = AB * AD and BC2 = BA * BD is appropriate.

3. The median removed from the end of the right angle of a right triangle is equal to half of the hypotenuse.

The following confirmation is also relevant.

4. If the median drawn on one side of a triangle is equal to half of that side, then such a triangle is right-angled, and the median drawn on the median is right-angled.

Since the proof of these assertions is so simple, we present only the proof of the third in several ways.

a) Let ABC be a triangle and let (Figure 2). On the AB side, we choose point D such that <DCB = and. In this case, the triangles ADC and BDC are equilateral. In this case CD is the median and CD = AB / 2.$< C = 90^o, < B = \alpha < A = 90^0 - \alpha\alpha < DCA = 90^0 - \alpha$

b) In triangle ABC we continue the CD median to a distance equal to it. The ACBC1 rectangle is formed (Figure 3).

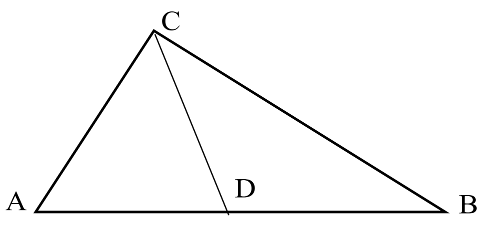


Figure 2

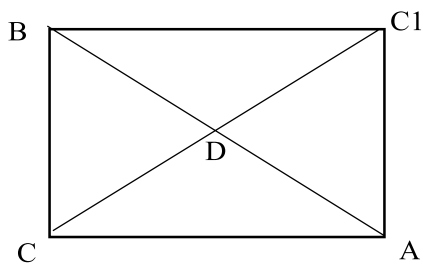


Figure 3

If such a process is performed on any triangle, a parallelogram is formed. The parallelogram in our examples consists of a right rectangle. So,

CD = 1/2 CC1 = 1/2 AB

3rd Problem. If an acute angle of a right triangle is 150, then prove that its hypotenuse is four times greater than the height subtended by that hypotenuse.

Proof. a) According to the problem condition <A = 150, we pass the CN median and CD - height from the end C, then <CND = 300, and since CD = ½ AB, CD = ¼ AB or AB = 4CD (Figure 4). Confirmation proved.

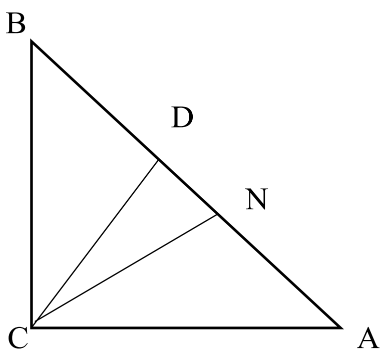


Figure 4

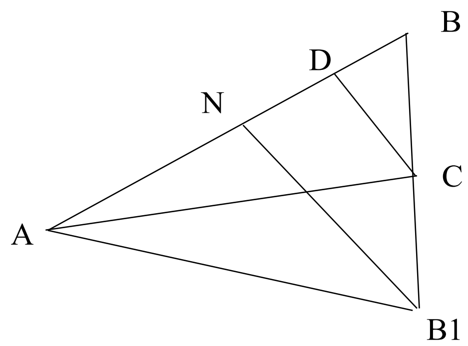


Figure 5

b) ΔABC (Figure 5). In this case, since BC = BC1, AB = AB1, <BAB1 = 300. Then we pass and pass, since B1 is N = 2CD and AB = AB1, so CD = 1 / 4AB and AB = 4CD.$\Delta ABB_1CD \perp ABB_1N \| DC$

4th Problem. The radius of the circle drawn inside a right-angled triangle is half the difference of the legs. Find the ratio of the large catheter to the small catheter.

Solution. In accordance with the terms of the case

$r = 1/2(a - b)$; $r = 1/2(a + b - \sqrt{a^2 + b^2}$

If we equate (ab) = a + b- then$\sqrt{a^2+b^2}$ $yoki\ a = b\sqrt{3}a: b = \sqrt{3}$

answer:$\sqrt{3}$

We now give a generalized case of the Pythagorean theorem.

To do this, we subtract the height of the CD from the hypotenuse AB of a right-angled triangle ABC. If we denote the linear elements of the resulting triangle CBD, ACD and ABC as P_a P_b and P_c, respectively, then$P_a^2 = P_b^2 + P_c^2$ appropriate.

The correctness of the last equation derives directly from the Pythagorean theorem.

5th Problem. The height reduced to the hypotenuse of a right-angled triangle makes it 2. If the radii of the circles drawn inside these 2 triangles are equal to 1 and 2, find the radius of the circle drawn inside the first triangle.

Solution. Similar linear elements of this triangle are r_a=1, r_b=2, and it is necessary to find r_c. (4) basically

$$r_c = \sqrt{r_a^2 + r_b^2} = \sqrt{1+4} = \sqrt{5}$$

Answer:$\sqrt{5}$

6th Problem. If the point D lies on the BC side of the triangle ABC, and BD = k CD = n and AD = d, then $d^2 a = b^2 k + c^2 n - ank$ proved to be.

Proof. We subtract AE Height from the A end of, so that if we apply the cosine theorem to the resulting triangles ABD and ADC, then (Figure 6).ΔABC

$$c^2 = d^2 + k^2 - 2kdcos < BDA,$$

$b^2 = d^2 + n^2 - 2dncos < ADC$ and

$ED = dcos < BDA$ because

$c^2 = d^2 + d^2 - 2kED$ (1) and

$c^2 = d^2 + n^2 - 2nED,$ (2) follows from the equations.

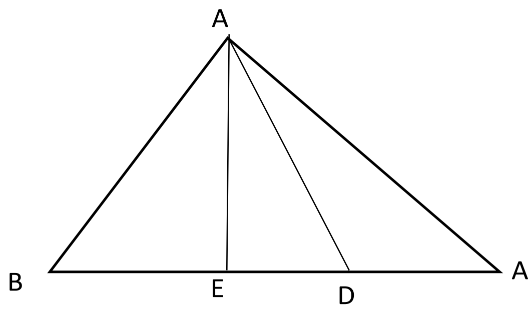


Figure 6

Then we multiply (1) by n and (2) by k and add:

$$c^2n + b^2k + nk(k + n) + d^2(k + n)$$

will be. It follows that k + n =. The confirmation has been proven. If BD: DC = p: q, then$ad^2a = b^2k + c^2n - ank$

$$d^2 = \frac{b^2p}{p+q} + \frac{c^2p}{p+q} - \frac{a^2qp}{(p+q)^2}$$

7th Problem. Given the sides of a triangle ABC, calculate the length of the bisector drawn on its side.a, b, ca

Since the solution is in triangle ABC AA1 = BA1: A1C = c: d and BA1 + A1C =$l_a a$

$BA_1 = \frac{a*c}{a+c}$ $va\ A_1C = \frac{a*c}{a+c}$arises (Figure 7)

Based on issue 6

$l_a^2 = \frac{b^2p}{p+q} + \frac{c^2p}{p+q} - \frac{a^2qp}{(p+q)^2}$ or

$$l^2 = bc\frac{(b+c)^2 - a^2}{(b+c)^2} \quad ; \quad 2p = a + b + c$$

if taken into account, then $l_a^2 = \frac{4bcp\ (p-a)}{(b+c)^2}$

bisectors drawn on such sides

$$l_a^2 = \frac{2}{b+c}\sqrt{bcp(p-a)}$$

$$l_a^2 = \frac{2}{a+c}\sqrt{acp(p-b)}$$

$$l_a^2 = \frac{2}{a+b}\sqrt{abp(p-c)}$$

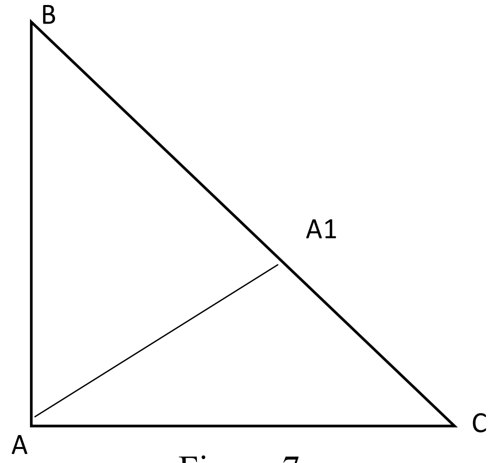


Figure 7

found using the formula.

8th Problem. From the acute angle of a right triangle, find the length of the bisector that divides its catheter into pieces p and q.

Solution. Let it be according to the condition of the matter (Figure 8). In that case$l_a = AD = x\Delta ADC$

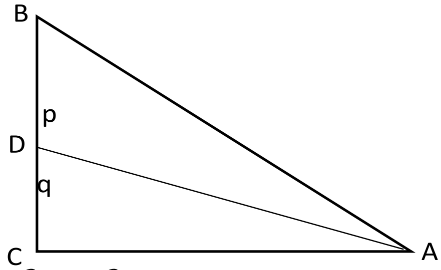
$$AC = \sqrt{x^2 - q^2}$$

and

$$AC = \sqrt{(p+q)^2 + x^2 - q^2}$$

because

$$\frac{(p+q)^2+(x^2-q^2)}{x^2-q^2} = \frac{p^2}{q^2}$$



$$\frac{(p+q)^2}{x^2-q^2} = \frac{p^2-q^2}{q^2}$$

$$\frac{p+q}{x^2-q^2} = \frac{p-q}{q^2}$$

Figure 8

$$X = AD = l_a = q\sqrt{\frac{2p}{p-q}}$$

9th Problem. ABC is for the point D obtained on the AB side of the triangle

$$AC^2 * BD^2 + BC^2 * AD^2 = AB^2 * CD^2$$

If the relation is reasonable, then this ABC triangle is right-angled. Prove that

Proof. If we enter the symbols AD = k, BD = n, CD = d in the triangle ABC.(Figure 9).

In that case, according to issue 6, $c^2d^2 = a^2r + +b^2n - knc$and according to the terms of the case:

$c^2d^2 = a^2k^2 + b^2 + n^2$ because $a^2k^2 + +b^2n^2 = a^2kc + b^2nc - knc$arises.

$(a^2 + b^2 - c^2)kn = 0$ or $a^2 + b^2 = c^2$we have equality.

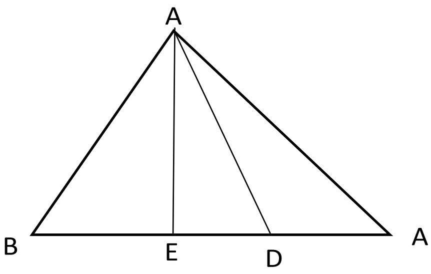


Figure 9

It turns out that.$< C = 90^{\circ}$

10th Problem. If the bisector of the right angle of a right triangle is proportional to the hypotenuse, then prove that the height lowered to the hypotenuse is it.$m: nm^2n^2$

Proof. Since the CD is conditionally bisector (Fig. 10)

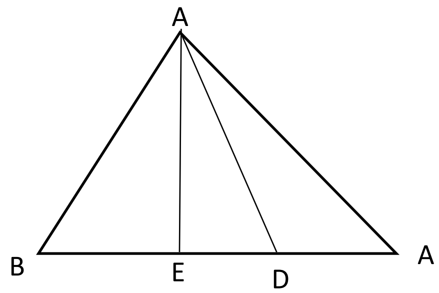


Figure 10

$AD: DB = m: n = AC: CB$known to be.

Since CE is the height:

$AC^2 = AE * AB$and so on$BC^2 = BE * AB AC^2: BC^2 = AE * BE = m^2n^2$

11th Problem. If the medians of the triangle ABC are proportional, then prove that the triangle is right-angled and find the sines of the acute angles.$m_a: m_{b:} m_c = \sqrt{1}: \sqrt{2}: \sqrt{3}$

Solution. To the formulas for calculating the medians as follows

$$4m_a^2 = 2b^2 + 2c^2 - a^2$$

$$4m_b^2 = 2a^2 + 2c^2 - b^2$$

$$4m_c^2 = 2a^2 + 2b^2 - c^2$$

and according to the condition of the case

$\frac{2b^2+2c^2-a^2}{2a^2+2c^2-b^2} = \frac{1}{2}$

$\frac{2b^2+2c^2-a^2}{2a^2+2b^2-c^2} = \frac{1}{3}$

write down

$4a^2 - 5b^2 - 2c^2 = 0$

$5a^2 - 4b^2 - 7c^2 = 0$

expressions. Then add them together to form an equation, from which the triangle <A = 900 is right-angled. Here$a^2 = b^2 + c^2$

$$m_a = k; \quad m_b = \sqrt{2}k; \quad m_a = \sqrt{3}k$$

Let's say, then

$$c = 2\sqrt{\frac{1}{3}}k \;\; ; \quad b = 2\sqrt{\frac{2}{3}}k \; ; \; a = 2k$$

we create

We find them.$sin < C = \frac{1}{\sqrt{3}}; \;\; sin < B = \sqrt{\frac{2}{3}}$

12th Problem. ABC protrudes from the ends of the triangle and corresponding its sides to A1, For each intersection AA1, BB1 and CC1 if the straight line intersecting at points B1, C1 intersects at point N inside the triangle

$$\frac{AN}{NA_1} = \frac{AB_1}{B_1C} + \frac{AC_1}{C_1B}$$

prove that the relationship is reasonable (Figure 11).

Proof. ABC is from the A end of the triangle

Convert BC // DE. In that case

$\Delta DNE \sim \Delta BNC \; va \; \Delta AND \sim \Delta A_1NC$

As a result

$$\frac{AN}{NA_1} = \frac{DE}{BC} = \frac{DA}{BC} + \frac{AE}{BC} (*)$$

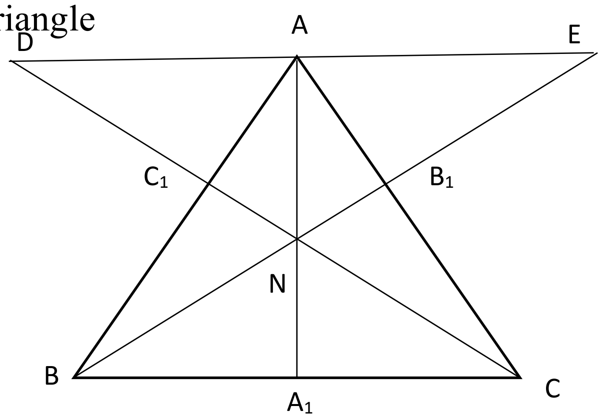


Figure 11

Equality arises.

$\Delta DAC_1 \sim \Delta CC_1B$ because

$\frac{DA}{BC} = \frac{AC_1}{C_1B}$ and $\Delta ABC_1 \sim \Delta CB_1B$

$\frac{AE}{BC} = \frac{AB_1}{B_1B}$ and $\frac{AN}{NA_1} = \frac{AC_1}{C_1B} + \frac{AB_1}{B_1C}$

It will be formed. Confirmation proved.

Prove that the following conclusions can be drawn from the assertion.

1) If, in particular, the intersections AA1, BB1, and CC1 consist of the medians of the triangles, then AN: NA1 = 2 is performed for each of them.

2) The bisectors of a triangle correspond at the point of intersection

$$\frac{b+c}{a}, \frac{a+b}{c} \quad va \quad \frac{a+c}{b}$$

divided into proportions.

13th Problem.A point N is obtained inside the triangle ABC, through which A1 A2 // BC, B1 B2 // AC and C1C2 // AB are passed, A1 A2: BC + B1 B2: AC + C1C2: Prove that the sum AB does not depend on the choice of point N (Figure 18).

Proof. Since A1 A2 // BC

because $\Delta AA_1A_2 \sim \Delta ABC$

$A_1A_2: BC = AA_1: AB$ equality and

since$\Delta BB_1B_2 \sim \Delta BAC$

equations$B_1B_2: AC = BB_1: AB$

is formed. Then

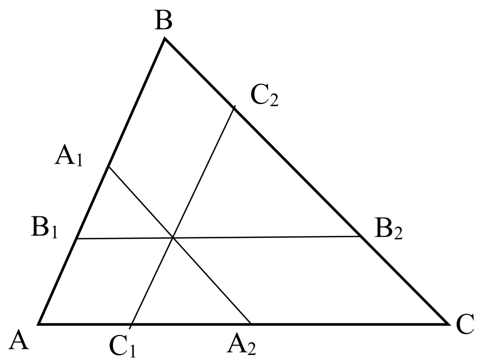


Figure 12

$$C_1C_2: AB = (C_1N + NC_2): AB = (AB_1 + A_1B): BC$$

Given that, we add the resulting equations:

$$\frac{A_1A_2}{BC}+\frac{B_1B_2}{AC}+\frac{C_1C_2}{AB}=\frac{AA_1}{AB}+\frac{BB_1}{AB}+\frac{AB_1+A_1B}{AB}=\frac{2AB}{AB}=2$$

The last equation means that the given sum is not dependent on N. Confirmation proved.

14th Problem. If there is an angle at the end of an equilateral triangle, prove that the relationship between the base and the side is reasonable.$\alpha=\frac{\pi}{7}a^4-3a^2b^2-ab^3+b^3=0$

Proof. According to the terms of the case

<A = a = p / 7 and <B = <C = 3p / 7

(Figure 13).

BD section <ABD = a condition

perform we spend

in which case <DBC = <BDC = 2a.

ABD and BCD are equilateral triangles

We can write cosa cos2a.$=\frac{\mathrm{b}}{2(\mathrm{b}-\mathrm{a})}=\frac{b-a}{2a}$

1 + cos2a = 2cos2a according to the relation $=\frac{b+a}{2a}\frac{b^2}{2(b-a)^2}$

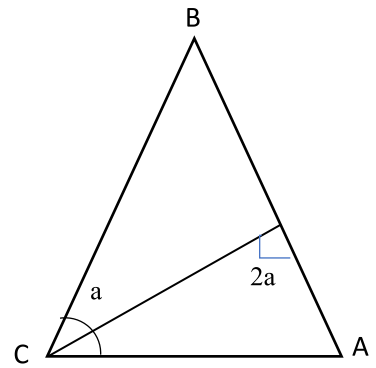


Figure 13

Or we create equality. Then we get the expression by multiplying both sides of the equation by a + b. Confirmation proved.$a^3-a^2b-2ab^2+b^3=0a^5-4a^3b^2-3ab^4+b^5=0$

Solving geometric problems develops students'thinking skills and increases their interest in the science of mathematics.

References

1.Azizkhojaeva N.N. Pedagogical technologies and pedagogical skills. Study guide. - T .: Publishing House of the Literary Foundation of the Writers' Union of Uzbekistan. 2006, 160 p.

2.Azamov A., Khaydarov B., Kuchkarov A., Sariqov Ye., Sagdiev U. Geometry. Textbook for 7th grade of general secondary schools. -T.: "Yangiyolpoligrafservis", 2017.

3.Rahimberdiev A. Geometry 8th grade. Textbook. - T .: "Teacher", 2014.

4.Haydarov B., Sariqov Ye., Kochkarov A. Geometry. 9th grade. - T .: "National encyclopedia of Uzbekistan", 2019.

5.Azamov A. Khaydarov B. The planet of mathematics. A handbook for organizing extracurricular activities in mathematics. -T .: "Teacher", 1993.